Drunken Heroine Quest

A Fantasy World Application of the Theory of Random Walks

Durham University

Introduction

Motivation

- Mathematics is fun, so is our research on random walk problems.
- The theory of random walks has a lot of useful applications in Physics, Biology or even some logistical problems.
- However, most of them are too serious for this poster.
- Instead, we will guide you through some basics of random walk theory, in a format of a fantasy story, a long long time ago...
- The idea of this application comes from a chat with my best friend Alphonse over an afternoon tea on a sunny day.

The story so far

- During the tea, Alphonse, as a novice game maker, told me he is thinking of the following plot for his new game:
- Once upon a time, the brave Edward went on a fearful quest of defeating a dragon to win the heart of the beautiful Dorothy.
- After falling foul of the curse of the Chief Warlock Albus, Edward is trapped in a skyscraping tower of unknown location in the boundless land of Hyrule.
- It is now up to Dorothy to break the curse to free her inamorato.
- With Edward nowhere to be found, alcohol seems to be the only way for Dorothy to pass the days.
- Without a particular direction nor a systematic search, a random walk journey starts.

Finding her love

The problem

- Alphonse then said he struggled to continue the story, because Dorothy may never see Edward again, especially because she needs to find the tower first.
- After a bit of thought, I replied to Alphonse that I may be able to help him with my research on random walks.
- Suppose for each step Dorothy walks, she picks a random direction, North, East, South or West, with equal probability, and walks with the same step size, regardless of the direction chosen.
- In the literature of Mathematics, this is called a simple symmetric random walk (SSRW) in a 2-dimensional lattice.
- Now we introduce the following theorem (amended into an easy word form) by a famous mathematician George Pólya in 1921.

Pólya's Recurrence Theorem

SSRW in d-dimensions is recurrent for $d = 1; 2$ and transient for $d > 2$.

- The word recurrent here means with probability one the walker would return to the starting point (or any other point).
- Transient means there is a positive probability that the walker will never return!
- Looking at Alphonse's confusing face, I tell him just to remember the following quote by another humorous mathematician Shizuo Kakutani, as it is somehow 'equivalent' to the theorem.

'A drunken man will find his way home, but a drunken bird may get lost forever.'

- So in conclusion, Dorothy will find the tower eventually.

Tower and moving variations

- 'So in what situation will Dorothy not find the tower, so that we can apply the second part of the theorem?' Alphonse asked.
- I was a bit surprised. It seems that he really did not want a good ending.
- 'Let me think, what if Dorothy is in fact an avian humanoid and the tower is on a floating island?'
- Now Dorothy can fly in six directions, including the four original directions and the two new directions up and down, with equal probability.
- This gives us a SSRW in a 3-dimensional lattice.
- Using the same theorem or the quote, we know with positive probability, Dorothy will never able to locate the tower.
- Alphonse (A): I see, after all it is not a very realistic model, because I cannot make a infinite size kingdom for my game! I may able put the world on a sphere though.
- Me, Hugo (H): That is even better, provided the land is finite, even if Dorothy can fly, using some basic properties of random walk we can show that Dorothy will still find the tower eventually.
- A: Wow, this is fascinating. In fact my ultimate goal is to let Dorothy to move or fly in any arbitrary direction, not only the fixed 4 or 6 directions. Would this affect the result?
- H: You have talents to be a mathematician! The model you suggested is in fact the continuous version of the simple model that I mentioned. It is related to Brownian motion if you know what that is. Anyway, the results should be still applicable with minor assumptions.

Ending

Further thoughts

- A: Mathematics is really fun and useful. However, your research must be much more complicated than this.
- H: Not much actually. using the same context of the story, I think about the journey of finding the tower, we also need Dorothy to swim across the Ocean of madness, pass through a labyrinth of trees and some underground passages in Wiggler Woods, maybe escape from a doppelganger emerged from her own shadow, and fight against a few more legendary evil creatures. Not much really.
- A: ...
- Oh, are you expecting the ending of the story?
- After a long long journey, Dorothy found and saved Edward. They escaped from the mystery tower, and live together happily ever after, with less alcohol, maybe.

Acknowledgements

The authors are grateful to Benjamin Chan Ming Hong for his technical advice on using the software Adobe InDesign and James McRedmond for the discussion on the topics of this poster.

References

[1] F. Comets, M.V. Menshikov, S. Volkov, and A.R. Wade, Random walk with barycentric self-interaction, J. Stat. Phys. 143 (2011) 855–888.

[2] K. Grill, On the average of a random walk, Statist. Probab. Lett. 6 (1988) 357–361.

[3] I.A. Ibragimov and Y.V. Linnik, Independent and Stationary Sequences of Random Variables, Wolters-Noordhoff, Groningen, The Netherlands, 1971.

Chak Hei Lo

Department of Mathematical Sciences, Durham University, UK.

c.h.lo@durham.ac.uk

Supervised by

Andrew R. Wade