Topology and Groups

Week 4, Thursday

1 Preparation

- 1.09 (Homotopy equivalence),
- 1.10 (Homotopy invariance).

2 Discussion

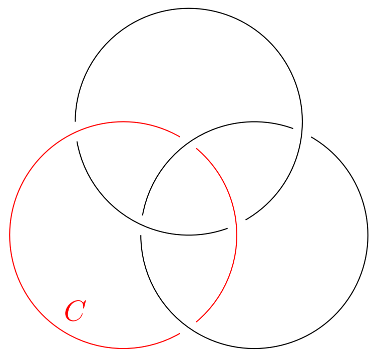
1. (PCQ) How many homotopy equivalence classes of symbols are there in the following list?

$$ABCDEFGHIJKLMNOPQRSTUVWXYZ0123456789$$

2. (PCQ) Show that homotopy equivalence is an equivalence relation.
3. Let K be the unit circle in the $(z = 0)$-plane in $\mathbf{R}^3$ (the *unknot*). Describe a 2-dimensional CW complex homotopy equivalent to $\mathbf{R}^3 \setminus K$. What is $\pi_1(\mathbf{R}^3 \setminus K)$?
4. What about $\mathbf{R}^3 \setminus (K_{-1} \cup K_1)$ where K_j is the unit circle in the $(z = j)$-plane $(j = -1, 1)$? What is $\pi_1(\mathbf{R}^3 \setminus (K_{-1} \cup K_1))$?
5. Find a 1-dimensional CW complex homotopy equivalent to the n-punctured plane. What is the fundamental group of the n-punctured plane?

3 Classwork

1. Consider the Borromean rings (below). These have the amazing property that any pair can be unlinked from one another, but all three cannot be simultaneously unlinked. Prove this, by expressing the red ring C as an element of the fundamental group of the complement of the other two and showing it is nontrivial.



2. Let L be the *Hopf link* pictured in the figure. Show that $\mathbf{R}^3 \setminus L$ is homotopy equivalent to $T^2 \vee S^2$ and hence compute the fundamental group of $\mathbf{R}^3 \setminus L$. Deduce that the Hopf link cannot be unlinked.