Topic 1.

$$\int_{\Gamma\backslash\mathbb{H}} u_j \, |E(z, \tfrac{1}{2}+it)|^2 \, d\mu(z)$$

$$\sim \int_{\Gamma\backslash\mathbb{H}} u_j \, E(z, \tfrac{1}{2}+it) \, E(z,s) \, d\mu(z)$$

$$\int_{\Gamma\backslash\mathbb{H}} E(\psi, z) \, |E(z, \tfrac{1}{2}+it)|^2 \, d\mu(z)$$

$$\|$$

$$\frac{1}{2\pi i} \int_{Re(s)=2} \hat{\psi}(s) \, E(z,s) \, ds$$

$$\int_{\Gamma\backslash\mathbb{H}} u_j(z) \, |u_n(z)|^2 \, d\mu(z)$$

$$\left| \int_{\Gamma\backslash\mathbb{H}} u_j(z) \, u_n(z) \, u_m(z) \, d\mu(z) \right|^2$$

$$\sim \frac{1}{216} \, \frac{\pi^4 \boxed{\Lambda(\tfrac{1}{2}, u_j \times u_n \times u_m)}}{\underbrace{\Lambda(1, sym^2 u_j)}_{\text{degree } 3} \Lambda(1, sym^2 u_n) \Lambda(1, sym^2 u_m)}$$

degree 8

Mathscinet 216 paper

subconvexity

2(a) $Re(s) \geq 1+\delta$

$1 \quad 1+\delta$

$J(\sigma + it) \geq K.$

$$\frac{1}{\zeta(\sigma+it)} \leq$$

$$\frac{1}{\zeta(s)} = \sum_{n\geq 1} \frac{\mu(n)}{n^s}, \qquad Re(s)>1$$

Möbius

$$\mu(n) = \begin{cases} 1 & n=1 \\ 0 & \text{if } n \text{ has a square divisor } p^2 \mid n \\ (-1)^r & n = \underbrace{p_1 p_2 \cdots p_r}_{\text{distinct}} \end{cases}$$

$$\left|\frac{1}{\zeta(\sigma+it)}\right| = \left|\sum \frac{\mu(n)}{n^{\sigma+it}}\right| \leq \sum_{n\geq 1} \frac{1}{n^\sigma}$$

$\sigma \geq 1+\delta$ $\qquad\qquad \zeta(\sigma) \leq \zeta(1+\delta)$

(b) $\frac{1}{\log t} \leq \zeta(1+it) \ll (\log t)^1$

$$\zeta(1+it) = \sum_{n\leq X} \frac{1}{n^s} + \frac{1}{(s-1)X^{s-1}} + \frac{\{X\}}{X^s}$$

$s = 1+it$
$$- s\int_X^\infty \frac{\{x\}}{x^{s+1}}\,dx$$

Take $X = [t] \sim t$

$$\left|\sum_{n\leq t} \frac{1}{n^s}\right| \leq \sum_{n\leq t} \frac{1}{n^1} \ll \log t$$

$$\left| \frac{\{t\}}{t^{1+it}} \right| \leq \frac{1}{t} = O(1)$$

$$s \int_X^\infty \frac{\{x\}}{x^{s+1}}\, dx \leq |1+it| \int_t^\infty \frac{1}{x^2}\, dx$$

$$\ll |t| \quad x^{-1} \Big|_{x=t}^{\infty} \quad \ll 1$$

$\mu(\sigma) = 0 \qquad \sigma \geq 1$

$$\mu(\sigma) = \inf \{\theta, \ |\zeta(\sigma+it)| \ll t^{\theta}\}$$

$\sigma > 1 \qquad |\zeta(\sigma+it)| \leq \zeta(\sigma)$

$\theta = 0$

$|\zeta(\sigma+it)| \gg 1$

$\forall \theta > 0$

$\sigma = 1 \qquad \frac{1}{\log t} \leq |\zeta(1+it)| \leq \log t \ll t^{\theta}$