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Rotational elasticity

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We consider an infinite three-dimensional elastic continuum whose material points experience no displacements, only rotations. This framework is a special case of the Cosserat theory of elasticity. Rotations of material points are described mathematically by attaching to each geometric point an orthonormal basis that gives a field of orthonormal bases called the coframe. As the dynamical variables (unknowns) of our theory, we choose the coframe and a density. We write down the general dynamic variational functional for our rotational theory of elasticity, assuming our material to be physically linear but the kinematic model geometrically nonlinear. Allowing geometric nonlinearity is natural when dealing with rotations because rotations in dimension three are inherently nonlinear (rotations about different axes do not commute) and because there is no reason to exclude from our study large rotations such as full turns.

The main result of the talk is an explicit construction of a class of time-dependent solutions that we call plane wave solutions; these are travelling waves of rotations. The existence of such explicit closed-form solutions is a non-trivial fact given that our system of Euler-Lagrange equations is highly nonlinear.

We also consider a special case of our rotational theory of elasticity which in the stationary setting (harmonic time dependence and arbitrary dependence on spatial coordinates) turns out to be equivalent to a pair of massless Dirac equations.

The talk is based on the paper [1].

[1] C.G.Boehmer, R.J.Downes and D.Vassiliev, Rotational elasticity, Quarterly Journal of Mechanics and Applied Mathematics, 2011, vol. 64, p. 415-439. The paper is a heavily revised version of preprint https://arxiv.org/abs/1008.3833